CSDA EXPLORATION OF DUO CURVATURE INHERENT WITH A UNIT PARABOLA GRAVITY FIELD RELATIVE TANGENT ENERGY SQUARE SPACE TIME CONSTRUCTION.

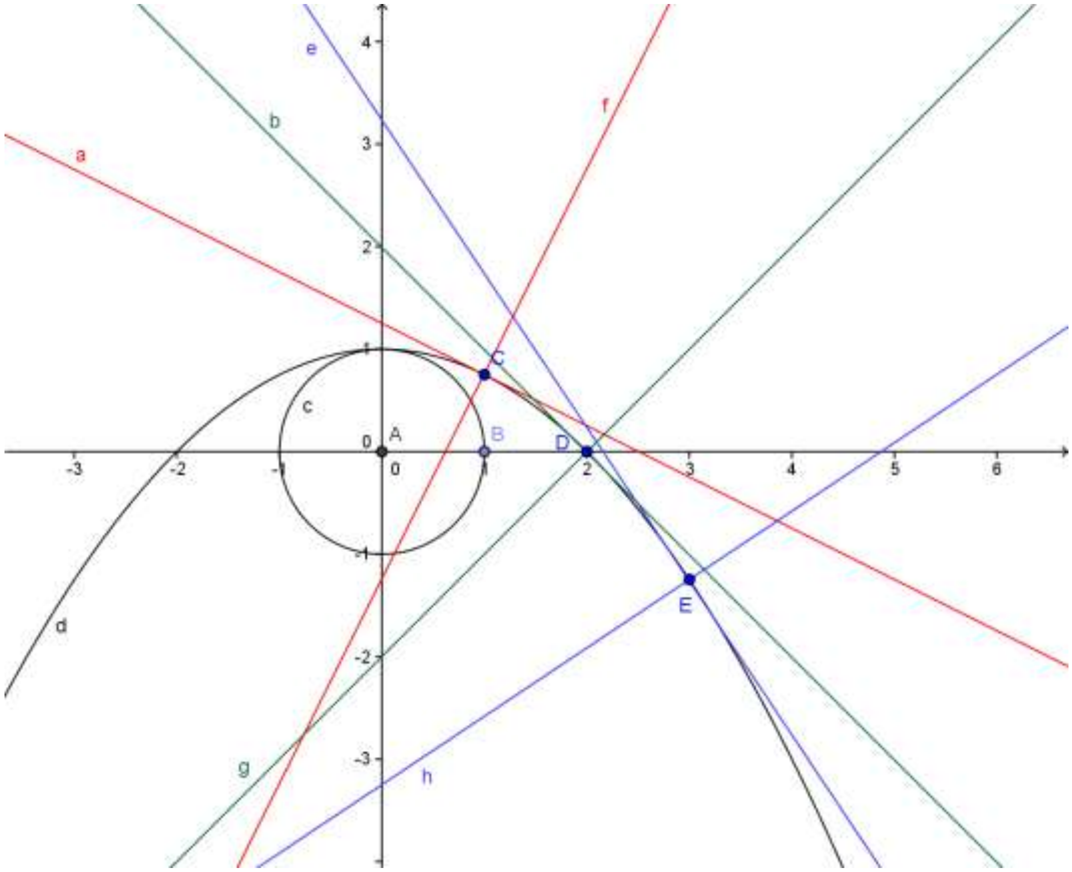
The previous demonstrations used a **CURVED SPACE DIVISION ASSEMBLY** to develop plane geometry methods of discovery concerning acceleration space curves. Math is inexorably connected with the physics of our being as are the curves of our being. Sir Isaac Newton's inverse square law will animate a **CSDA** by defining system initial curvature of spin diameters and using an **RT** to make congruent captured rotation energy curves with system field spin phenomena defines by initial **PRINCIPAL ASI** curvature. Only after separating differential geometry definition of curvature from a time and energy **CSDA** definition of curvature, can we animate the physical energy properties of curves.

Both geometries (Differential and CSDA) use curve framing to rationalize the relativity between lines and curves. First step in discovery of radius of curvature is constructing a tangent to the point considered. Next we build a right angle frame at the considered point with a tangent normal.

Curvature is a number only. To realize a linear magnitude for length, we inverse the numerical value of curvature. A tangent normal holds this linear definition of curvature as radius of an osculating circle kissing and nestled at the point of considered loci.

The next two demonstrations reveal the dual curvature of a Sand Box Geometry unit parabola. It is necessary to visit Calculus curvature concepts to understand differential geometry position curvature and curvature attendant with time/energy inverse square motion curves. I will be using three points on a unit parabola locus: (1, 3/4), (2, 0), and (3, -5/4), to analyze plane geometry meter of both position and motion curvature.

Sand Box Geometry demonstration of curve framing used to ascertain a differential definition of loci radius of curvature (a Geogebra computer construction).



Methods to conduct curve analysis of analytic geometry point composition.

- Conduct curve frame discovery of curvature for unit parabola (d) about unit circle (c) at parabola loci points (C, D, and E).
- Construct tangents (a, b, and e) for points (C, D, and E).
- The differential geometry radius of definition for curvature evaluation of each point lies on the tangent normal (f, g, and h)

The method to evaluate curvature of a point will be found in any first year calculus text book. Let (κ) be the curvature of a point on our parabola loci; then:

$$k = \frac{|2nd derivative|}{(1 + (first derivative)^2)^{\frac{3}{2}}}$$

In plain language the above term refers to a fraction. The numerator is an absolute value of the second derivative. The denominator refers to a sum (1+the first derivative squared) raised to the 3/2 power. If we only worked with circles, this entire operation means curvature of a circle would be the inverse of its radius.

circle	Radius in units	curvature
A	2	$\frac{1}{2}$
B	$\frac{1}{2}$	2
C	3	$\frac{1}{3}$
D	$\frac{1}{3}$	3

Curvature is a number only. Radius is magnitude and controls size of curvature. I will use *Mathematica* to demonstrate the following identity of a parabola focal radius at locus position (1, ¾). The focal radius of a Sand Box Geometry **CSDA** provides method to evaluate a radius resultant for loci curvature of position *and* central force energy curves.

$$(vertexradiusofcurvature) \times (focalradius) \times \left(\sqrt{focalradius}\right) = \left[\frac{|f''(r)|}{\left(1+\left(f'(r)\right)^2\right)^{\frac{3}{2}}}\right]^{-1}$$

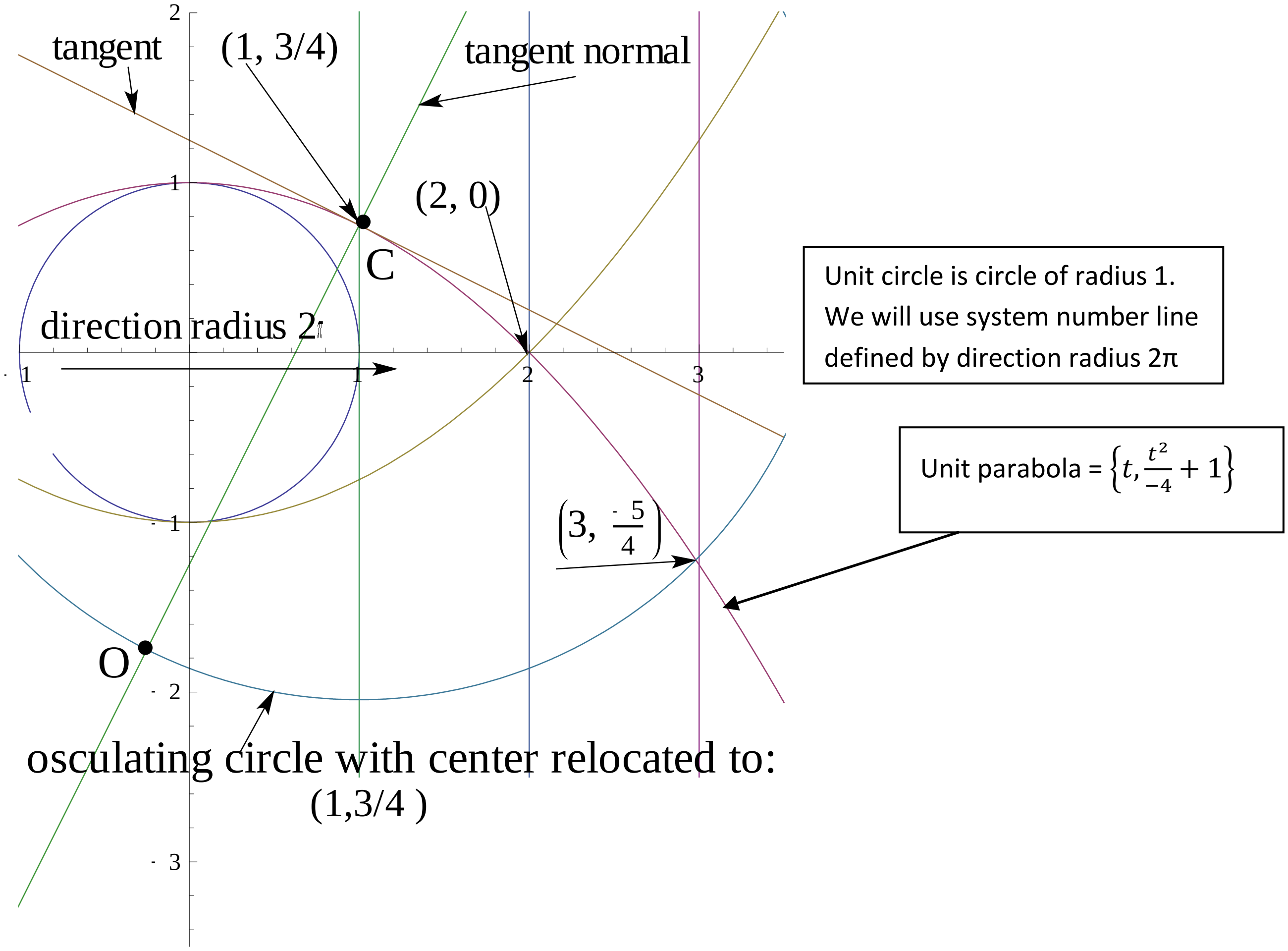
Parametric instructions to construct a unit circle and unit parabola, locus point (1, ¾), tangent and tangent normal, needed to study curvature of this point will be:

$$\text{ParametricPlot}[\{\{1\text{Cos}[t],1\text{Sin}[t]\},\{t,\frac{t^2}{-4}+1\},\{t,\frac{t^2}{+4}-1\},\{1,t\},\{2,t\},\{3,t\},$$

$$\{t,(\frac{5-2t}{4})\},\{t,(\frac{-5+8t}{4})\},\{\frac{5\sqrt{5}}{4}\text{Cos}[t]+1,\frac{5\sqrt{5}}{4}\text{Sin}[t]+\frac{3}{4}\}\},\{t,\frac{-5}{2},\frac{7}{2}\},$$

$$\text{PlotRange}\to\{\{-1,\frac{7}{2}\},\{\frac{-7}{2},2\}\}]$$

Osculating circle radius is line $(\overleftrightarrow{OC})$.



Object Identification:

- $\{1\text{Cos}[t], 1\text{Sin}[t]\}, \{t, \frac{t^2}{-4} + 1\}, \{t, \frac{t^2}{+4} - 1\}$→ Unit Circle and North and South Unit Parabola.
- $\{1, t\}, \{2, t\}, \{3, t\}$→ Abscissa component of points (C, D, E) being (1, ¾), (2, 0), and (3, -5/4).
- $\{t, (\frac{5-2t}{4})\}, \{t, (\frac{-5+8t}{4})\}$, → Tangent and tangent normal at (1, ¾).
- $\{\frac{5\sqrt{5}}{4}\text{Cos}[t] + 1, \frac{5\sqrt{5}}{4}\text{Sin}[t] + \frac{3}{4}\}$→ Osculating circle defining radius of curvature for point (1, ¾).

An osculating circle will use its center and radius of that center to a loci point to define magnitude of curvature. As such, the arc of the circle should be touching the tangent at (1, ¾). If we move the center of an osculating circle to the point, then we can see the size of the radius of curvature at the intercept of the tangent normal and osculating arc. Plane geometry definition of curvature for parabola locus at (1, ¾) is osculating radius$(\overline{OC})$. Notice the osculating radius $(\overline{OC})$ has nothing to do with beginning center zero of the system, and only demonstrate differential geometry determination for radius of curvature of a locus point.

The next demonstration will use differential calculus to determine curvature of points (1, 3/4), (2, 0) and (3, -5/4) for parabola loci points C, D, and E, as comparative with **CSDA** focal radius evaluation.

Finding radius of curvature for points (C, D, and E) and focal radius identity:

I will use a *Mathematica* template to determine curvature.

$$\text{point C } (1,\frac{3}{4}) \xrightarrow{\text{yields}} \frac{\text{Abs}[\frac{-1}{2}]}{(1+(\frac{t}{-2})^2)^{\frac{3}{2}}} /. t \to 1 \xrightarrow{yields} \frac{4}{5\sqrt{5}}$$

$$\text{point D } (2,0) \xrightarrow{\text{yields}} \frac{\text{Abs}[\frac{-1}{2}]}{(1+(\frac{t}{-2})^2)^{\frac{3}{2}}} /. t \to 2 \xrightarrow{yields} \frac{1}{4\sqrt{2}}$$

$$\text{point E } (3,\frac{-5}{4}) \xrightarrow{\text{yields}} \frac{\text{Abs}[\frac{-1}{2}]}{(1+(\frac{t}{-2})^2)^{\frac{3}{2}}} /. t \to 3 \xrightarrow{yields} \frac{4}{13\sqrt{13}}$$

These evaluations are curvature only and must be inversed to find radius of curvature. Using a Sand Box Geometry **CSDA** focal radius gives us a radius of curvature magnitude directly. This next table assembles comparative data. Then we can move to a Geogabra demonstration on unit circle unit parabola central relativity.

parabola loci	$\{1, \frac{3}{4}\}$	$\{2, 0\}$	$\{3, \frac{-5}{4}\}$	
differential ID	$\left(\frac{4}{5\sqrt{5}}\right)^{-1}$	$\left(\frac{1}{4\sqrt{2}}\right)^{-1}$	$\left(\frac{4}{13\sqrt{13}}\right)^{-1}$	
focal radius ID	$\frac{5\sqrt{5}}{4}$	$4\sqrt{2}$	$\frac{13\sqrt{13}}{4}$	
focal radius magnitude	$\frac{5}{4}$	2	$\frac{13}{4}$	

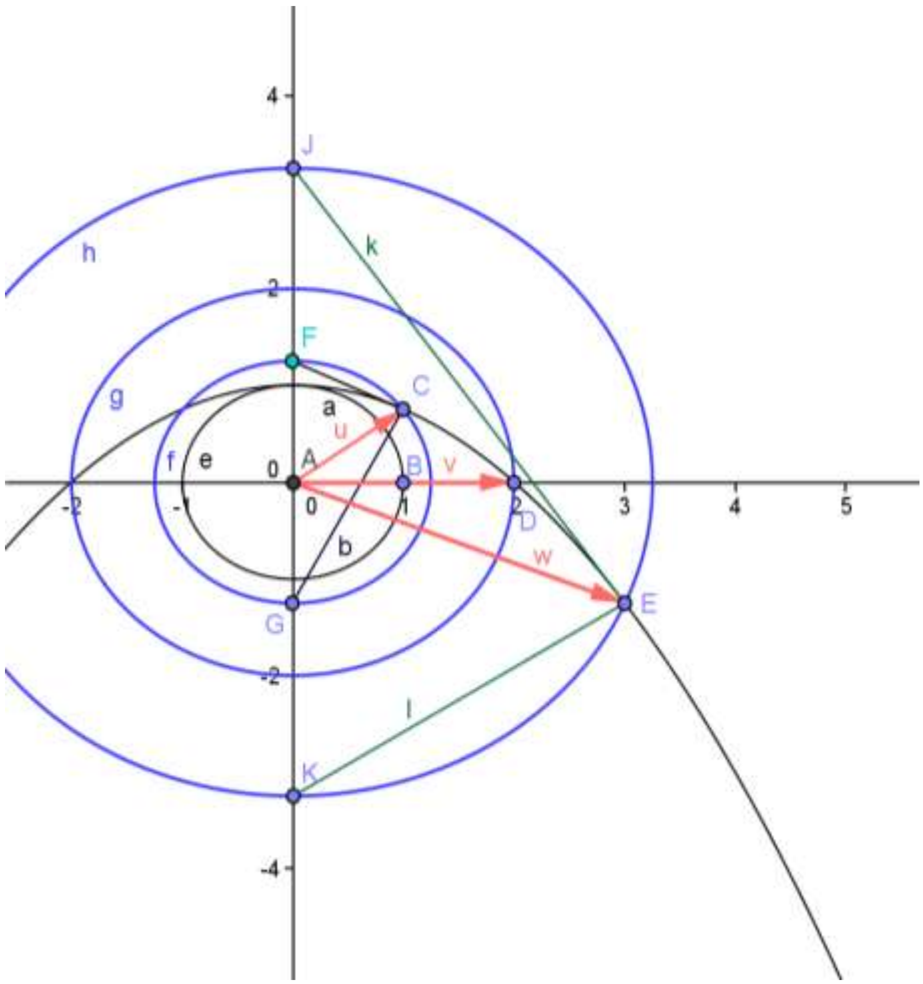
All parabola vertex radius of curvature is 2(*p* = *r*) of the unit circle.

$$point\ C\ \left(1,\frac{3}{4}\right) \xrightarrow{yields} \left(2*\frac{5}{4}*\sqrt{\frac{5}{4}}\right) \xrightarrow{\text{yields}} \frac{5\sqrt{5}}{4}$$

$$point\ D\ (2,0) \xrightarrow{yields} (2*2*\sqrt{2}) \xrightarrow{yields} 4\sqrt{2}$$

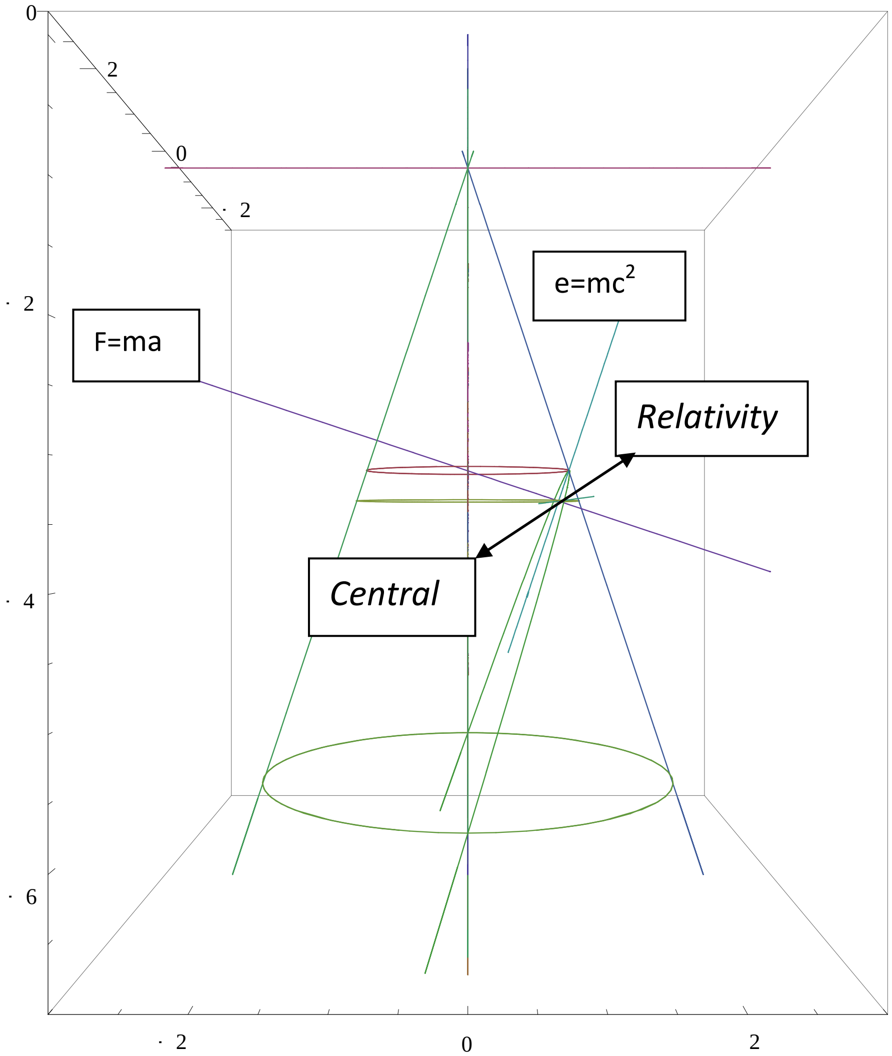
$$point\ E\ (3,\frac{-5}{4}) \xrightarrow{yields} \left(2*\frac{13}{4}*\sqrt{\frac{13}{4}}\right) \xrightarrow{yields} \frac{13\sqrt{13}}{4}$$

A changing focal radius of a unit parabola is one and the same as changing central radii of the unit circle. I will show the parabola loci tracks changing curvature of iterate circle/sphere with a Geogebra construction.



We see that focal radii grow outward from the central unit circle as *initial* curvature of consideration toward radii of *final* concentric curvature of consideration. In the above **CSDA**; focal radius (u) points to a right angle vertex of curve framing for position analysis of differential geometry at point C *and* also defines a right triangle spin diameter for our physical world displaying an expanding central relative spherical wave form passing through C. The same can be said for focal radius (v) to point D, and focal radius (w) to point E.

The unit circle center point (0) does have a dual identity. One is traditional central position of a Cartesian coordinate system (zero). The other identity will animate curved space phenomena of a Sand Box Geometry **CSDA** when we allow center (0) to become **F** of a natural central force. Be it a stone tossed in water or expanding energy of a super nova, a focal radius will follow moving expanding energy curves. **END SAND BOX GEOMETRY POSITION AND ENERGY CURVES**.



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